

# Heat transfer and pressure drop characteristics in rectangular channels with elliptic pin fins

Qingling Li, Zhong Chen <sup>\*</sup>, Ulrich Flechtner, Hans-Joachim Warnecke

*Department of Chemical Engineering, University of Paderborn, 33098 Paderborn, Germany*

Received 1 February 1997; accepted 25 November 1997

## Abstract

Experiments have been carried out to investigate the heat transfer and flow resistance characteristics in rectangular ducts with staggered arrays of short elliptic pin fins in a crossflow of air. By employing the heat/mass transfer analogy and the naphthalene sublimation technique, the mean heat transfer coefficients on pin fins and on the endwall (base plate) of the channel have been presented, respectively. The total mean heat transfer coefficients of pin fin channels are calculated and the resistance coefficients are also investigated. The experimental results show that the heat transfer of a channel with elliptic pin fins is somewhat higher than that with circular pin fins while the resistance of the former is much lower than that of the latter in the Reynolds number range from 1000 to 10 000. © 1998 Elsevier Science Inc. All rights reserved.

**Keywords:** Elliptic pin fins; Heat transfer; Pressure drop; Heat/mass transfer analogy

## Notation

$2a$	major axis length of pin fin (m)
$2b$	minor axis length of pin fin (m)
$a_t$	thermal diffusivity ( $\text{m}^2/\text{s}$ )
$A$	mass transfer area ( $\text{m}^2$ )
$A_p$	mass transfer area of a pin fin ( $\text{m}^2$ )
$A_w$	mass transfer area on the endwall ( $\text{m}^2$ )
$c_w$	concentration of naphthalene vapour on surface ( $\text{kg}/\text{m}^3$ )
$c_i$	concentration of naphthalene vapour in air ahead of the test section ( $\text{kg}/\text{m}^3$ )
$c_e$	concentration of naphthalene vapour in air behind the test section ( $\text{kg}/\text{m}^3$ )
$\Delta c$	logarithmic mean concentration (—)
$D_\pi$	equal circumference diameter ( $D_\pi = U/\pi$ ) (m)
$D_{AB}$	mass diffusion coefficient ( $\text{m}^2/\text{s}$ )
$E$	fin efficiency (—)
$h$	pin fin length (m)
$\Delta m$	mass loss (kg)
$\Delta p$	pressure drop ( $\text{N}/\text{m}^2$ )
$Q$	volume flow rate ( $\text{m}^3/\text{s}$ )
$S1$	spanwise pitch (m)
$S2$	streamwise pitch (m)
$S_p$	cross section area of a pin fin ( $\text{m}^2$ )
$u$	maximum flow velocity ( $\text{m}^2/\text{s}$ )
$U$	circumference of pin fins (m)
$Z$	total number of pin fin rows (—)

## Greek

$\alpha$	total heat transfer coefficient ( $\text{Wm}^{-2}\text{K}^{-1}$ )
$\alpha_p$	heat transfer coefficient on pin fins ( $\text{Wm}^{-2}\text{K}^{-1}$ )
$\alpha_w$	heat transfer coefficient on the endwall ( $\text{Wm}^{-2}\text{K}^{-1}$ )
$\beta$	mass transfer coefficient (m/s)
$\beta_p$	mass transfer coefficient on pin fins (m/s)
$\beta_w$	mass transfer coefficient on endwall (m/s)
$\lambda$	thermal conductivity of air ( $\text{Wm}^{-1}\text{K}^{-1}$ )
$\lambda_p$	thermal conductivity of pin fin material ( $\text{Wm}^{-1}\text{K}^{-1}$ )
$\rho$	density of air ( $\text{kg}/\text{m}^3$ )
$\tau$	sublimating time (s)
$\mu$	dynamic viscosity (Pa s)
$\nu$	kinematic viscosity ( $\text{m}^2/\text{s}$ )

## Dimensionless numbers

Eu	Euler number ( $\text{Eu} = \Delta p/(\rho u^2 Z)$ )
Nu	Nusselt number ( $\text{Nu} = \alpha D_\pi/\lambda$ )
Pr	Prandtl number ( $\text{Pr} = \nu/a_t$ )
Re	Reynolds number ( $\text{Re} = u D_\pi/\nu$ )
Sc	Schmidt number ( $\text{Sc} = \nu/D_{AB}$ )
Sh	Sherwood number ( $\text{Sh} = \beta D_\pi/D_{AB}$ )

## 1. Introduction

Short pin fins are widely used in many industrial applications, especially in the trailing edges of gas turbine blades, in some modern electronic systems and in aerospace industry. There have been many investigations for heat transfer and

<sup>\*</sup>Corresponding author. Address: Department of Chemical Engineering, Wuhan Institute of Chemical Technology, Wuhan 430073, People's Republic of China.

the pressure drop of narrow channels with short pin fins (Armstrong et al., 1988; Lau et al., 1989; Babus'Haq et al., 1995), but most prior works are restricted to pin fins with circular cross section. The research on pin fins with other cross sections is relatively sparse. Sparrow et al. (1991) investigated experimentally the pressure drop characteristics of diamond-shaped pin fin arrays, which were used on the space shuttle flight STS-34 launched on 18 October 1989. In another article, Granis et al. (1991) presented the numerical simulation of fluid flow through an array of diamond-shaped pin fins. Metzger et al. (1984) studied the heat transfer and pressure drop characteristics for an array of oblong pin fins at various attack angles. Their experiments show that heat transfer of oblong pin fin arrays is higher by approximately 20% over that of corresponding arrays with round pin fins, but these increases are offset by increases in pressure loss of about 100%.

It is well known that tubes or cylinders with slim or streamline-shaped cross section have much less flow resistance than circular ones. On the other side, it is by no means clear that circular pin fins are the best for heat transfer. In fact, long cylinders with slim cross section sometimes have better heat transfer characteristics. Ota et al. (1984) studied the heat transfer and flow around an elliptic cylinder of axis ratio 1:3. Their experimental results show that heat transfer coefficients of the elliptic cylinder is higher than that of a circular one with equal circumference and the pressure drag coefficients of the former are much lower than that of the latter. Li et al. (1996) have recently investigated the convective heat transfer and pressure drop for arrays of drop-shaped cylinders in crossflow. Their conclusions show that the mean heat transfer coefficients of drop-shaped cylinder arrays are about 8–29% higher than those of

the corresponding circular cylinder arrays, and the pressure drop of the former is only about half that of the latter. Therefore, it is essential to investigate further the pin fins with slimmer cross sections in order to enhance heat transfer and decrease the flow resistance.

The objective of the present study is to investigate the heat transfer and pressure drop characteristics in a narrow channel with staggered arrays of elliptic pin fins. By means of the heat/mass transfer analogy and the naphthalene sublimation technique, the heat transfer coefficients on pin fins and on the end-wall (base plate) were measured separately in order to find out the contribution of each to the total heat transfer of the pin fin channel.

## 2. Experimental apparatus and technique

### 2.1. Wind tunnel

The suction-mode wind tunnel used in this experiment is shown in Fig. 1. The test section has a cross section 240 mm wide and 12.75 mm high and was made from 10 mm thick plexiglass. The blower is connected to the wind tunnel with a flexible tube in order to reduce the mechanical vibration of the system.

### 2.2. Instrumentation

The flow velocity in the wind tunnel is measured with a rotameter (see Fig. 1). The pressure drop has been achieved

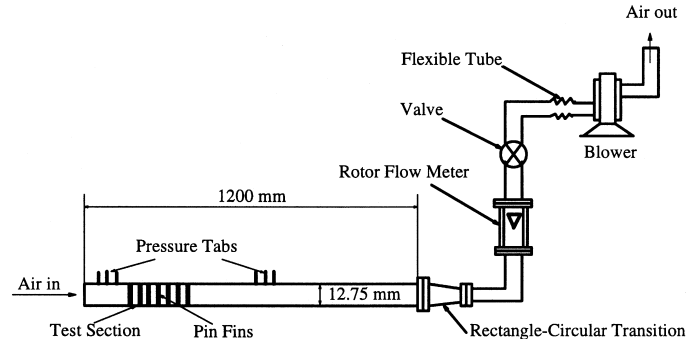


Fig. 1. Wind tunnel system.

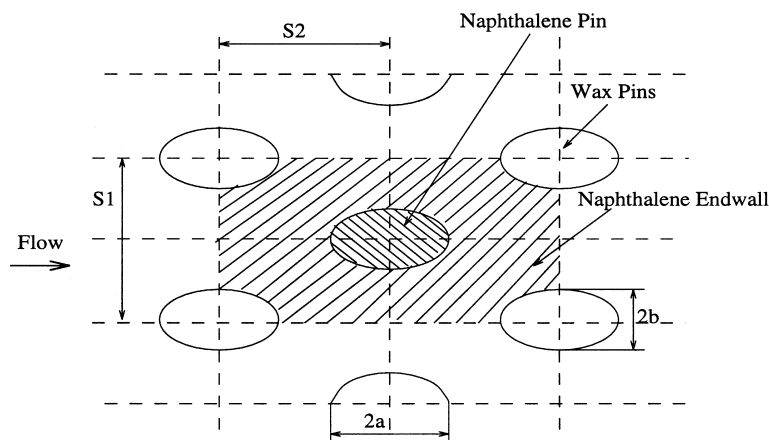


Fig. 2. Configuration of elliptic pin fins and their arrangement.

through measuring the static pressure ahead of and behind the test section by means of pressure taps together with micro-manometer. An analytical balance was used to measure the mass loss during a test run and air temperature in the wind tunnel is measured both with thermocouples and with standard glass thermometers.

### 2.3. Experimental specimens

The configuration of the elliptic-shaped pin fins and their arrangement are as shown in Fig. 2. Its major axis length  $2a = 16$  mm and the minor axis length  $2b = 9$  mm. Thus the equal-circumference-diameter of pin fins is  $D_\pi = 12.75$  mm (see Eq. (1)). The height of pin fins is also 12.75 mm (equal to the height of the wind tunnel). The relative spanwise and streamwise pitches are  $S1/D_\pi = 1.10$ – $3.00$ . In the experiments only one pin fin is made of naphthalene for each run and the others are made of wax.

By manufacturing the pin fins, the melted analytical naphthalene (or wax) was poured into a specially designed steel mould that had been painstakingly fabricated in order to provide casting with very smooth surfaces. The pin fin specimens are formed when they cool down.

The test specimens for heat/mass transfer on endwall are also manufactured by casting. The mould for the naphthalene specimens is shown in Fig. 3.

### 2.4. Experimental technique and procedures

The naphthalene sublimation technique is employed in the present research. That is, by means of measuring the mass loss before and after a test run the mean heat transfer coefficients can be achieved by heat/mass transfer analogy. According to its symmetry, the local modeling method is adopted in the experiment and the mass transfer is confined to the oblique line area of Fig. 2. The pin fin made of naphthalene is always placed in the middle of the 10 rows pin fin arrays for every test run. This means, the experimental results can be used for fully developed flow cases and not for the arrays with only a few pin fin rows.

## 3. Data reduction

All Reynolds numbers in this paper are based on the equal-circumference-diameter  $D_\pi$  of the elliptic pin fins and velocities  $u$  calculated using the minimum flow area in pin fin arrays, therefore

$$D_\pi = U/\pi, \quad (1)$$

$$Re = D_\pi u/\nu, \quad (2)$$

where  $U$  is the circumference of an elliptic cylinder. The mean mass transfer coefficient is defined as

$$\beta = \frac{\Delta m}{At\Delta c}, \quad (3)$$

where  $\Delta m$  is the sublimating mass during the test time  $\tau$  on the sublimating surface area  $A$  and the logarithmic mean concentration difference is

$$\Delta c = \frac{(c_w - c_i) - (c_w - c_e)}{\ln[(c_w - c_i)/(c_w - c_e)]}, \quad (4)$$

where  $c_w$  is the naphthalene vapour concentration on sublimating surface and can be gotten by naphthalene vapour pressure equation (Sogin et al., 1958) together with the ideal gas law. The  $c_i$  and  $c_e$  are the naphthalene vapour concentrations in the air ahead of and behind the test section in the wind tunnel. In the present work we have

$$c_i = 0, \quad (5)$$

$$c_e = \frac{\Delta m}{\tau Q}. \quad (6)$$

Thus the average Sherwood number of the pin fins is

$$Sh_p = \beta_p D_\pi / D_{AB} \quad (7)$$

and the Sherwood number on endwall is

$$Sh_w = \beta_w D_\pi / D_{AB}. \quad (8)$$

According to heat/mass transfer analogy, we have

$$Nu_p = \frac{\alpha_p D_\pi}{\lambda} = \left(\frac{Pr}{Sc}\right)^n Sh_p, \quad (9)$$

$$Nu_w = \frac{\alpha_w D_\pi}{\lambda} = \left(\frac{Pr}{Sc}\right)^n Sh_w, \quad (10)$$

where  $n = 0.37$  for turbulent flow (Zukauskas, 1972).

The total mean Nusselt number of the duct with pin fins can be calculated as

$$Nu = \frac{\alpha D_\pi}{\lambda} = \frac{(Nu_p A_p E + Nu_w A_w)}{A_p + A_w}, \quad (11)$$

where  $A_p$  and  $A_w$  are the effective heat transfer surface area on pin fins and on endwall, respectively.  $E$  is the fin efficiency

$$E = \frac{\tanh(mh)}{mh}, \quad (12)$$

where  $h$  is the height of pin fins and  $m$  can be calculated as

$$m = \left[ \frac{\alpha_p U}{\lambda_p S_p} \right]^{1/2}, \quad (13)$$

where  $\lambda_p$  is the thermal conductivity of pin fins and  $S_p$  is the cross section area of a pin fin.

The Euler number for representing the pressure drop is defined as

$$Eu = \frac{\Delta p}{\rho u^2 Z}, \quad (14)$$

where  $\Delta p$  is the overall pressure drop of the test section and  $Z$  is total row number of pin fin arrays.

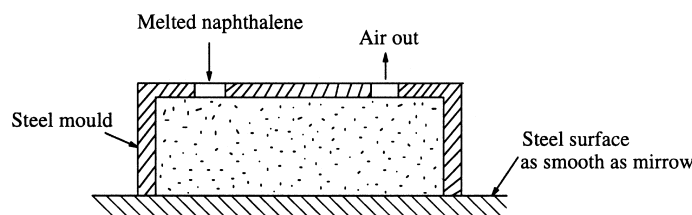


Fig. 3. Mould for casting specimens of endwall.

#### 4. Experimental results and discussion

Heat transfer of pin fins at both arrangements is shown in Fig. 4. The effect of the pin fin arrangement on mean  $Nu_p$  is quite strong. The compact pin fin array ( $S1/D_\pi = S2/D_\pi = 1.1$ ) has higher heat transfer coefficients than the other at the same Reynolds number. For the two arrangements, the experimental data in Reynolds number range from 1000 to 9000 can be well represented by two least squares fits.

$$Nu_p = 0.279Re^{0.615} \quad \text{for } S1/D_\pi = S2/D_\pi = 1.10, \quad (15)$$

$$Nu_p = 0.322Re^{0.544} \quad \text{for } S1/D_\pi = S2/D_\pi = 3.00. \quad (16)$$

Fig. 5 displays the effect of arrangement of pin fins on mean heat transfer of the endwall. The experimental results can be also expressed as the following

$$Nu_w = 0.096Re^{0.733} \quad \text{for } S1/D_\pi = S2/D_\pi = 1.10, \quad (17)$$

$$Nu_w = 0.099Re^{0.676} \quad \text{for } S1/D_\pi = S2/D_\pi = 3.00. \quad (18)$$

Fig. 6 presents the comparison between Nusselt numbers of pin fins and those on the endwall of the channel. For both arrangements, the heat transfer rate on the pin fins is higher than that on the endwall at the same Reynolds number. The difference at the lower Reynolds numbers is quite large but with the increase of  $Re$  the difference decreases quickly. For example, for the more compact array,  $Nu_p$  is about 22.6% higher than  $Nu_w$  at  $Re = 1500$  but only 2.24% at  $Re = 7000$ .

The total mean Nusselt numbers of the channel with elliptic pin fins are shown in Fig. 7. Here the fin efficiency is taken into account and the material of the pin fins is assumed to be steel. The correlation of the results is

$$Nu = 0.462Re^{0.530} \quad \text{for } S1/D_\pi = S2/D_\pi = 2.50. \quad (19)$$

The mean heat transfer data (average  $Nu$  of row 5–10) of a channel with circular pin fins presented by Metzger et al. (1982) is also displayed in Fig. 7 for comparison. It is clear seen that the heat transfer coefficients of ducts with elliptic pin fins are higher than those with circular ones.

Finally, Fig. 8 shows the mean resistance for each row. The pin fin array used for friction experiment has 10 rows. A least squares fit to the results gives

$$Eu = 1.960Re^{-0.340} \quad \text{for } S1/D_\pi = S2/D_\pi = 2.50. \quad (20)$$

It can be seen from Fig. 8 that the pressure drop of channels with elliptic pin fins is much lower than the corresponding ducts with circular ones. For example, the former is only about 58% and 44% of the latter at  $Re = 1200$  and  $Re = 8000$ , respectively.

#### 5. Conclusions

1. The channel with elliptic pin fins has better heat transfer characters than that with circular ones in the Reynolds number range tested.

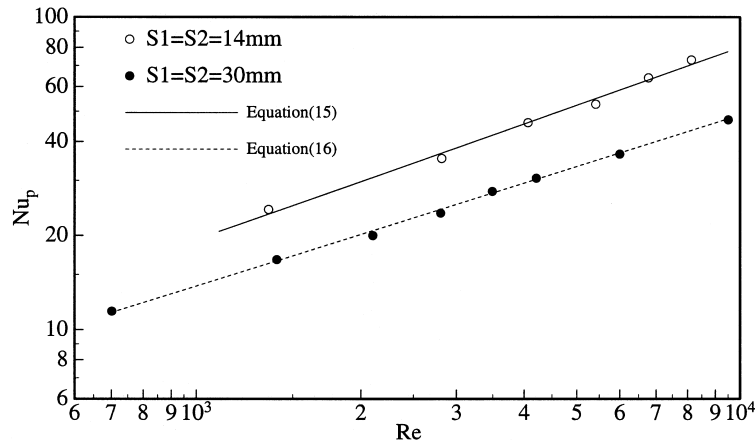


Fig. 4.  $Nu_p$  versus  $Re$ .

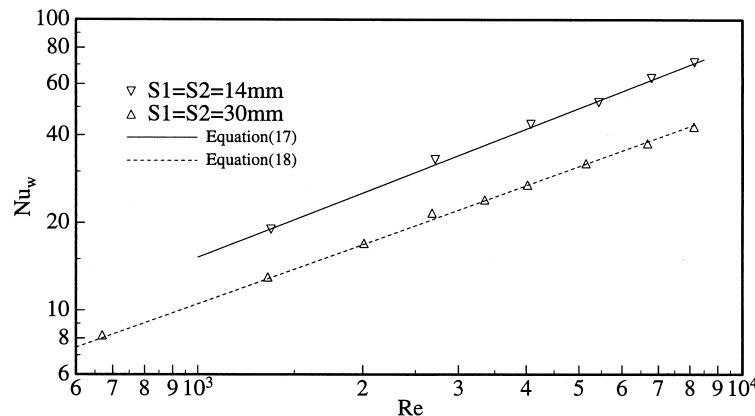


Fig. 5.  $Nu_w$  versus  $Re$ .

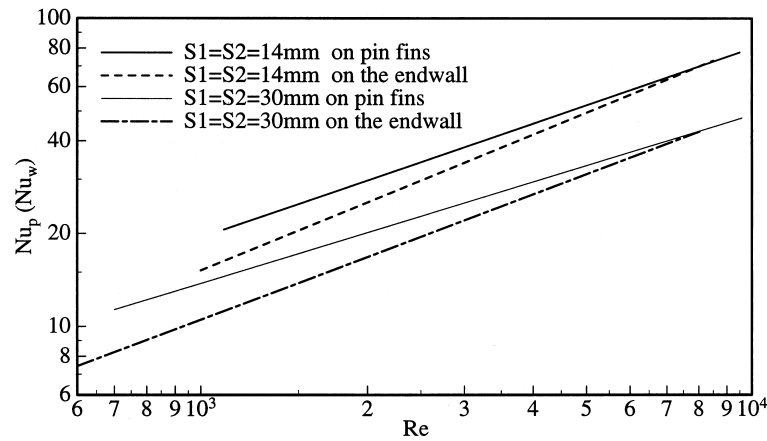


Fig. 6. Comparison of the heat transfer rates between pin fins and endwall.

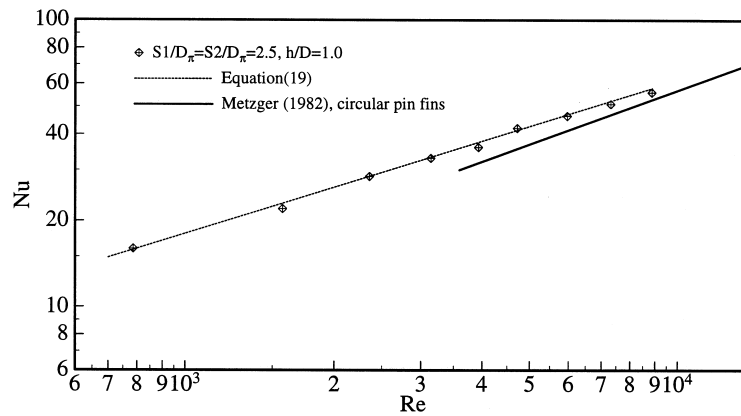


Fig. 7. Comparison of Nu with Metzger et al. (1982).

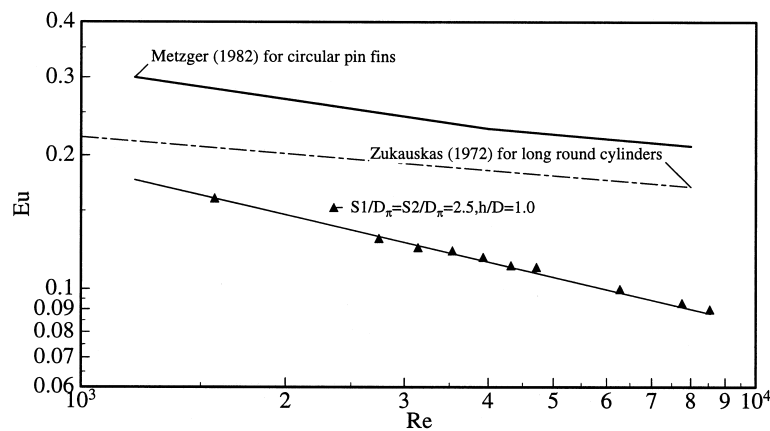


Fig. 8. Eu versus Re.

2. Over the whole Reynolds number range of interest, the elliptic pin fin channel has much lower flow resistance than that with circular pin fins. The Euler numbers of the former are only about 58%–44% of the latter when Re changes from 1200 to 8000.

3. The heat/mass transfer coefficients on pin fins and on the endwall can be easily measured separately with the naphthalene sublimation method.

4. The Nusselt numbers for the pin fins are generally higher than those of the endwall in the Re range from 700 to 8000.

5. The arrangement of pin fins affects both heat transfer of pin fins and that of the endwall. The more compact pin fins arrays have higher heat transfer rate for the relative pitches investigated.

## References

- Armstrong, J. et al., 1988. A review of staggered array pin fin heat transfer for turbine cooling applications, *J. Turbomachinery* 110, 94–103.
- Babus'Haq, R.F. et al., 1995. Thermal performance of pin fin assembly, *Int. J. Heat and Fluid Flow* 16, 50–55.
- Grannis, V.B. et al., 1991. Numerical simulation of fluid flow through an array of diamond-shaped pin fins, *Numer. Heat Transfer* 19, 381–403.
- Lau, S.C. et al., 1989. Turbulent heat transfer and friction in pin fin channels with lateral, flow ejection, *ASME J. Heat Transfer* 111, 51–58.
- Li, Q.L. et al., 1996. Konvektive wärme/stoffübertragung und druckverlust in rohrbündeln bestehend aus tropfenförmigen Rohren, *Chemie Ingenieur Technik* 68, 1299–1302.
- Metzger, D.E. et al., 1982. Developing heat transfer in rectangular ducts with staggered arrays of short pin fins, *ASME J. Heat Transfer* 104, 700–706.
- Metzger, D.E. et al., 1984. Effects of pin shape and array orientation on heat transfer and pressure loss in pin fin arrays, *ASME J. Eng. for Gas Turbines and Power* 106, 252–257.
- Ota, T. et al., 1984. Heat transfer and flow around an elliptic cylinder, *Int. J. Heat Mass Transfer* 27 (10), 1771–1779.
- Sogin, H.H. et al., 1958. Sublimation mass transfer through compressible boundary layers on a flat plate, *Trans. ASME*, January, 61–69.
- Sparrow, E.M. et al., 1991. Pressure drop characteristics of heat exchangers consisting of arrays of diamond-shaped pin fins, *Int. J. Heat Mass Transfer* 34, 589–600.
- Zukauskas, A.A., 1972. Heat transfer from tubes in crossflow. *Adv. Heat Transfer* 8, 93–160.